Published by Institute of Physics Publishing for SISSA

Received: *January 12, 2006*
Revised: *February 15, 2006*
Accepted: *February 16, 2006*
Published: *March 1, 2006*

Non-supersymmetric attractors and entropy function

Mohsen Alishahiha and Hajar Ebrahim

Institute for Studies in Theoretical Physics and Mathematics (IPM)
P.O. Box 19395-5531, Tehran, Iran
E-mail: alishah@theory.ipm.ac.ir, ebrahimi@theory.ipm.ac.ir

Abstract: We study the entropy of non-supersymmetric extremal black holes which exhibit attractor mechanism by making use of the entropy function. This method, being simple, can be used to calculate corrections to the entropy due to higher order corrections to the action. In particular we apply this method for five dimensional non-supersymmetric extremal black hole which carries two magnetic charges and find the R^2 corrections to the entropy. Using the behavior of the action evaluated for the extremal black hole near the horizon, we also present a simple expression for C-function corrected by higher order corrections.

Keywords: Black Holes, Black Holes in String Theory.

Contents

1. Introduction

One of the interesting features of the black hole in $\mathcal{N} = 2$ supersymmetric theories is the attractor mechanism [1 − 3]. In this mechanism the values of different scalar fields at the horizon are determined entirely by the charges carried by the black hole regardless of their values at asymptotic infinity. Another important feature of the $\mathcal{N} = 2$ supersymmetric theories is that the theory can be described by a function so called *prepotential*. Actually by making use of this rich structure of the supersymmetric black hole, one can study different aspects of these black holes such as the entropy.

In fact the entropy of 1/2-BPS black holes in $\mathcal{N} = 2$ supersymmetric string theories in four dimensions has been calculated which is in agreement with microscopic counting of the states of the corresponding brane configurations representing these black holes [4 − 8].

Recently it has been shown that the Legendre transformation of the entropy with respect to the electric charges of the black hole is related to the prepotential. This observation has led to a conjecture relating the entropy to the partition function of topological string theory [9 − 14]. It could also give a practical way to compute the entropy of the black hole.

So far we have considered only the cases where the theory is supersymmetric. One may wonder if there is similar structure for non-supersymmetric black holes as well. Actually it has been shown that the attractor mechanism can also work for non-supersymmetric extremal black holes [15 − 22]. Having had the attractor mechanism one would like to know whether this can be used to study different aspects of non-supersymmetric black holes such as entropy.

More recently it has been shown [23] that the similar structure as what we have in the supersymmetric black hole appears in the non-supersymmetric case as well. This observation has provided a simple method to compute the entropy of a spherically symmetric extremal black hole in a theory of gravity coupled to abelian gauge fields as well as neutral scalar fields with arbitrary higher order derivative interactions. In this method one first defines the *entropy function* which is a function of the parameters labeling the near horizon background. By extremizing this function with respect to the parameters one can find the values of these parameters in terms of the black hole charges. Moreover the entropy of the black hole is equal to this function at the extremum.[1]

To be precise let us consider an extremal D-dimensional black hole whose near horizon geometry is $AdS_2 \times S^{D-2}$ and carries electric and magnetic charges. We have also several scalar fields. One can define the entropy function by the Lagrangian density evaluated for this background integrated over $(D-2)$-sphere. The Legendre transformation of this function with respect to the electric charges, evaluated at the extremum, is equal to the entropy of the black hole divided by 2π. In this sense the entropy function plays the role of the prepotential in the supersymmetric case. This method has been used to compute corrections to the entropy of the different black holes due to higher order corrections to the effective action [26, 27]. It is the aim of this article to further study this method for non-supersymmetric black holes which exhibit attractor mechanism.

The organization of the paper is as follows. In section 2 we shall develop the procedure of evaluating the entropy function for those extremal black holes which exhibit the attractor mechanism. Using this method we will reproduce the results of [17]. In section 3 we will compute corrections to the entropy due to higher order corrections to the action using entropy function. We will consider a particular form of the higher order corrections, namely the Lovelock type action. In section 4 we shall consider a specific model in which the equations can exactly be solved. In five dimensions this model can be treated as a toy model for the five dimensional black hole considered in [28]. We will also compute R^2 corrections to the entropy and find the same structure as that in the supersymmetric case which has recently been computed[2] [29]. The last section is devoted to the discussions where we present different aspects of the models we are considering. In particular we see how one can naturally defined a C-function for the models we are studying and how this function gets corrections due to higher order corrections to the action.

2. Entropy function for non-supersymmetric attractor

In this section we shall study extremal black hole solution in D-dimensions whose metric has the following form

$$ds^2 = G_{\mu\nu}dx^\mu dx^\nu = -a^2(r)dt^2 + \frac{dr^2}{a^2(r)} + b^2(r)d\Omega_{D-2}^2, \tag{2.1}$$

where $a^2(r)$ has double zero at horizon. This black hole solution will also be supported by some scalar fields as well as electric and magnetic fields.

[1] We note that similar function has also been found in the supersymmetric case in [24, 25]. Although in the supersymmetric case is was not called entropy function, it has precisely the same preoperties.

[2] The entropy of 5-dimensional black hole in the presence of higher order corrections was also studied in [30 – 32]

One might think about this as a solution of an effective action which typically could have the following form

$$I = \frac{1}{\kappa_D^2} \int d^D x \sqrt{-G} \left(R - 2(\partial\phi_i)^2 - f_{ab}^{(e)}(\phi_i) F_a^{(e)} F_b^{(e)} - f_{\alpha\beta}^{(m)}(\phi_i) F_\alpha^{(m)} F_\beta^{(m)} + \cdots \right), \quad (2.2)$$

where dots stand for higher order corrections. Of course we note that, in general, adding higher order terms to the action could change the shape of the metric, nevertheless we will assume that the isometry of the metric remains unchanged though the metric components might get corrections. The only point which is crucial for our consideration is that $a^2(r)$ has double zero at the horizon. In other words, we are interested in the extremal black hole solution.

We would like to study the entropy one can associate with this black hole. These kinds of black holes and their properties including entropy have recently been studied in [17] in the context of non-supersymmetric attractors where the authors showed that these solutions exhibit the attractor mechanism by which several moduli fields are drawn to fixed values at the horizon of the black hole regardless of the values they take at asymptotic infinity. It has also been shown that the entropy of these black holes is given in terms of these values at horizon.

Here we shall use another approach to find the entropy of these extremal black holes. Following [23] we will first define entropy function which leads to a simple method to evaluate the entropy of the black hole. Being simple, this method can also be used to find the corrections to the entropy when we are taking into account higher order corrections to the action.

To proceed we shall use the general formula for the entropy in the presence of higher derivative terms which has been studied in [33 − 37]. In our case, taking into account that the covariant derivatives of the tensor fields are zero, we get

$$S_{BH} = 8\pi \int d^{D-2}x \sqrt{G_{D-2}}\, \frac{\partial \mathcal{L}}{\partial R_{rtrt}} g_{rr} g_{tt}\,. \quad (2.3)$$

As it was shown in [23] one may find a simple expression for the entropy of the black hole, using a particular rescaling of the coordinates. To be specific let us consider the following extremal black hole solution of the action (2.2) [17]

$$ds^2 = -a^2(r)dt^2 + \frac{dr^2}{a^2(r)} + b^2(r)d\Omega_{D-2}^2, \qquad F_{D-2}^\alpha = p^\alpha \sqrt{\omega_{D-2}}, \quad (2.4)$$

where ω_{D-2} is the determinant of unit $(D-2)$-sphere. We also have a non-zero scalar filed ϕ.

To find the entropy function and thereby the entropy one considers an ansatz for the extremal black hole solution at near horizon as follows

$$ds^2 = v_1 \left(-a^2(r)dt^2 + \frac{dr^2}{a^2(r)} \right) + v_2 b^2(r) d\Omega_{D-2}^2, \qquad F_{D-2}^\alpha = p^\alpha \sqrt{\omega_{D-2}}, \quad (2.5)$$

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We note that in our notation one has $R_{trtr} = -\frac{a^2(r)''}{2v_1} g_{tt} g_{rr}$, which can be used to write the entropy formula as the following

$$S_{BH} = -2\pi \frac{2}{a^2(r)''} \int d^{D-2}x\sqrt{-G}\, \frac{\partial \mathcal{L}}{\partial R_{\mu\nu\rho\sigma}} R_{\mu\nu\rho\sigma}, \tag{2.6}$$

for $\mu, \nu, \rho, \sigma = r, t$, where $a^2(r)'' = \frac{d^2 a^2(r)}{dr^2}$.

Let us define $\mathcal{L}_\lambda$ to be the same as the original Lagrangian except that each factor of R_{rtrt} in the expression of the $\mathcal{L}$ is multiplied by a factor of λ. Thus one can rewrite the above equation as

$$S_{BH} = -2\pi \frac{2}{a^2(r)''} \int d^{D-2}x\sqrt{-G}\, \frac{\partial \mathcal{L}_\lambda}{\partial \lambda}\bigg|_{\lambda=1} = -\frac{4\pi}{a^2(r)''} \frac{\partial f_\lambda(\vec{p}, v_i, \phi)}{\partial \lambda}\bigg|_{\lambda=1}, \tag{2.7}$$

where the function f is defined by

$$f(\vec{p}, v_i, \phi) = \int d^{D-2}x\sqrt{-G}\, \mathcal{L}, \tag{2.8}$$

and the right hand side is evaluated in the near horizon geometry (2.5). Here we have assumed that the solution has a horizon at $r = r_H$ where $a^2(r_H) = a^2(r_H)' = 0$ and the above expression has ultimately to be evaluated at $r = r_H$. On the other hand the other parameters such as v_i can be found using their equations of motion. We note, however, that f is a function of r which in our procedure it is taken as a fixed parameter. In fact as we shall see the r-dependence will be dropped in the final expression for the entropy. Actually since we are interested in the solutions which exhibit attractor mechanism, near the horizon one may assume that $a \approx 1 - (r_H/r)^{D-3}$. Therefore near the horizon we can define a new coordinate $\hat{r} = r - r_H$ such that $a \approx \frac{(D-3)\hat{r}}{r_H}$. In this coordinates system the solution near the horizon is $AdS_2 \times S^{D-2}$.

Using the same argument as [23] one can see that the function f_λ must be of the from $v_1 \tilde{f}_\lambda(\vec{p}, \phi, v_2, \lambda v_1^{-1})$ and thus we arrive at

$$\lambda \frac{\partial f_\lambda(\vec{p}, \vec{v}, \phi)}{\partial \lambda} + v_1 \frac{\partial f_\lambda(\vec{p}, \vec{v}, \phi)}{\partial v_1} - f_\lambda(\vec{p}, \vec{v}, \phi) = 0. \tag{2.9}$$

Therefore setting $\lambda = 1$ in (2.9) and using the equations of motion one can find an expression for the entropy in terms of the function f as follows

$$S_{BH} = -\frac{4\pi}{a^2(r)''}\, f. \tag{2.10}$$

Here the left hand side is evaluated at the near horizon when the parameters v_i and ϕ are fixed using their equations of motion which can be given by extremizing f with respect to the corresponding parameters

$$\frac{\partial f}{\partial \phi} = 0, \qquad \frac{\partial f}{\partial v_i} = 0. \tag{2.11}$$

Here we assume that these equations have a solution.

Actually this is a special case of what considered in [23] where the entropy is found to be the Legendre transformation of function f, defined as (2.8), with respect to the electric charges. But in our case since the solution carries only magnetic charge the first term in the Legendre transformation is zero. We will come back to this point later.

To see how this formalism works let us compute f for the solution (2.5). Doing so we arrive at

$$f = \frac{\Omega_{D-2}}{16\pi} v_1 v_2^{\frac{(D-2)}{2}} b^{D-2} \left(\frac{(D-3)(D-2)v_1 - 2(D-3)^2 v_2}{v_1 v_2 b^2} - \frac{(D-2)! V_{\text{eff}}}{v_2^{D-2} b^{2(D-2)}} \right), \tag{2.12}$$

where $V_{\text{eff}} = f_{\alpha\beta}(\phi) p^\alpha p^\beta$. In writing this equation we have used the fact that a^2 has double zero root at horizon and also $b^2(r) a^2(r)'' = 2(D-3)^2$ in the near horizon limit. The later assumption can be obtained directly from the equations of motion. Now we need to extremize f with respect to v_i and ϕ

$$\frac{\partial f}{\partial v_1} = 0, \qquad \frac{\partial f}{\partial v_2} = 0, \qquad \frac{\partial f}{\partial \phi} = 0. \tag{2.13}$$

The last equation leads to $\partial_\phi V_{\text{eff}} = 0$ which we assume that this equation as a solution at $\phi = \phi_0$. From the first two equations one finds

$$v_1 = v_2 = \left(\frac{(D-4)!\, V_{\text{eff}}}{b^{2(D-3)}} \right)^{\frac{1}{D-3}}. \tag{2.14}$$

Evaluating f at these values we get

$$f = -2 \frac{\Omega_{D-2}}{16\pi} \frac{(D-3)^2}{b^2} \Big((D-4)!\, V_{\text{eff}}(\phi_0) \Big)^{\frac{D-2}{2(D-3)}}. \tag{2.15}$$

Therefore the entropy is given by

$$S = \frac{\Omega_{D-2}}{4} \Big((D-4)!\, V_{\text{eff}}(\phi_0) \Big)^{\frac{D-2}{2(D-3)}}. \tag{2.16}$$

It is worth noting that the r-dependence is dropped in the final result and the entropy is given by the value of the potential at its extremum which is given in terms of the magnetic charge and $f_{\alpha\beta}(\phi_0)$. We note also that by plugging v_1 and v_2 into the metric, one can read the radius of horizon in terms of the effective potential

$$r_H^2 = \Big((D-4)!\, V_{\text{eff}}(\phi_0) \Big)^{1/(D-3)}, \tag{2.17}$$

in agreement with [17].

3. Higher order corrections to entropy function

In this section we shall study corrections to the entropy due to higher order corrections to the action. In general adding higher order corrections can change the entropy in two different ways. The corrections could be due to the additional terms in the action when we are evaluating f using the zeroth order solution for the metric and other fields. One may also consider the case where the modification is due to the fact that adding these terms would change the equations of motion and thereby change the solution. We note that in the procedure we use to compute the entropy both effects will be taken into account.

To be specific we consider higher order corrections to the action to be of the Lovelock type where the higher order corrections are given by the extended Gauss-Bonnet action [38]

$$I = \frac{1}{\kappa^2}\int d^D x\sqrt{G}\sum_{m=1} \frac{\lambda_m}{2^m}\,\delta^{\rho_1\sigma_1\cdots\rho_m\sigma_m}_{\mu_1\nu_1\cdots\mu_m\nu_m}\,R^{\mu_1\nu_1}{}_{\rho_1\sigma_1}\cdots R^{\mu_m\nu_m}{}_{\rho_m\sigma_m}, \tag{3.1}$$

where λ_m are free parameters which could be either constant or one may consider the case where they depend on the scalar fields. $\delta^{\rho_1\sigma_1\cdots\rho_m\sigma_m}_{\mu_1\nu_1\cdots\mu_m\nu_m}$ is totally antisymmetric product of m Kronecher deltas, normalized to take values ± 1. For $m = 1$ setting $\lambda_1 = 1$ we get standard Einstein action, for $D = 2m$ the mth term is topological and for D dimensional space-time all terms for $m > D/2$ identically are equal to zero.

The corrections to the entropy of the small black hole in the heterotic string theory due to this action has recently been studied in [27] where the near horizon solution is $AdS_2 \times S^{D-2}$. It is the aim of this section to study these corrections for the extremal black hole solution given by (2.4) using the procedure we have developed in the previous section.

To find the entropy we need to calculate function f for the solution (2.5) using the above action. Actually it was shown [39] that for the metric of type

$$ds^2 = g_{ab}dx^a dx^b + r^2(x)d\Omega^2_{D-2}, \qquad a, b = 1, 2, \tag{3.2}$$

one gets the following expression for the Gauss-Bonnet densities integrated over the unit $(D-2)$-sphere

$$\begin{aligned}\frac{1}{\kappa^2}\int d^{D-2}x\sqrt{-G}\mathcal{L}_m = &-\frac{\Omega_{D-2}}{\kappa^2}\frac{(D-2)!}{(D-2m)!}\lambda_m\sqrt{-g}\, r^{D-2m-2}[1-(\nabla r)^2]^{m-2}\\ &\times\Big\{2m(m-1)r^2[(\nabla_a\nabla_b r)^2-(\nabla^2 r)^2]\\ &\quad +2m(B-2m)r\nabla^2 r[1-(\nabla r)^2]-mr^2\mathcal{R}[1-(\nabla r)^2]\\ &\quad -(D-2m)(D-2m-1)(1-(\nabla r)^2]^2\Big\},\end{aligned} \tag{3.3}$$

where $\mathcal{R}$ is the 2-dimensional Ricci scalar of the metric g_{ab}. Plugging the solution (2.5) into the equation (3.3) and taking into account that all terms which have covariant derivative vanish for the near horizon solution, one can find f as follows

$$\begin{aligned}f = \frac{\Omega_{D-2}v_1}{16\pi}\Big\{&-\frac{(D-2)!\,V_{\rm eff}}{v_2^{(D-2)/2}b^{D-2}}+\sum_{m=1}^{[D/2]}\frac{(D-2)!}{(D-2m)!}\lambda_m v_2^{(D-2m)/2}b^{D-2m}\\ &\times\left(\frac{(D-2m)(D-2m-1)v_1-mv_2b^2a^2(r)''}{v_1v_2b^2}\right)\Big\}.\end{aligned} \tag{3.4}$$

For $m = 1$ setting $\lambda_1 = 1$ we get the leading order action studied in the previous section, the other terms can be considered as higher order corrections to the action. We still assume that $a^2(r)$ has double zero root at the horizon which means that although it might get corrections from higher order terms in the action the $a^2(r)''$ remains a non-zero constant at the horizon. Actually this is the only assumption we make in the rest of this section. Now the same as before we will consider b (or r) as a fixed parameter and extremize f with respect to v_i and ϕ. Then the entropy will be given by (2.10) at this extremum.

To get an insight how this procedure works, let us study some examples explicitly. In four dimensions we have only $m = 1, 2$ and so we get

$$f = \frac{1}{4}\left(2v_1 - v_2 b^2 a^2(r)'' - \frac{2v_1}{v_2 b^2} V_{\rm eff}\right) - \lambda_2 a^2(r)'', \tag{3.5}$$

from which one finds $v_1 = a^2(r_H)'' V_{\rm eff}(\phi_0)/2,\ v_2 = V_{\rm eff}(\phi_0)/b^2$ and therefore we arrive at

$$S = \pi V_{\rm eff}(\phi_0) + 4\pi\lambda_2. \tag{3.6}$$

Note that the entropy is corrected by a constant due to Gauss-Bonnet term. On the other hand using this result and plugging them into the metric one can read the radius of the horizon which is given by $r_H^2 = V_{\rm eff}$ that is the same as the leading order which we have studied in the previous section. We note, however, that λ_2 could be a function of ϕ, *i.e.* $\lambda_2 = g(\phi)\tilde{\lambda}_2$. In this case although the expression for r_H in terms of the effective potential is the same as the leading order, it could get corrections due to the corrections of ϕ_0 which in this case it is the solution of $\partial_\phi V_{\rm eff}(\phi) = -4\tilde{\lambda}_2 \partial_\phi g(\phi)$.

In five dimensions we still have $m = 1, 2$ and the function f reads

$$f = \frac{\pi}{8}\left(b\sqrt{v_2}(6v_1 - v_2 b^2 a^2(r)'') - \frac{6v_1}{v_2^{3/2} b^3} V_{\rm eff}\right) - \frac{3}{2}\pi\tilde{\lambda}_2 g(\phi)\sqrt{v_2}\, b a^2(r)'', \tag{3.7}$$

which upon extremizing it with respect to v_1 and v_2 one gets

$$v_2 = \frac{\sqrt{V_{\rm eff}(\phi_0)}}{b^2}, \qquad v_1 = \frac{a^2(r)''}{8}\left(\sqrt{V_{\rm eff}(\phi_0)} + 4\tilde{\lambda}_2 g(\phi_0)\right), \tag{3.8}$$

where ϕ_0 is a solution of $\partial_\phi f = 0$. The entropy is given by

$$S = \frac{\pi^2}{2}\left(V_{\rm eff}^{3/4}(\phi_0) + 12\tilde{\lambda}_2 g(\phi_0) V_{\rm eff}^{1/4}(\phi_0)\right). \tag{3.9}$$

It is worth noting that to get consistent results one needs to assume that $\partial_\phi f = 0$ has a solution. Since at leading order the function f is proportional to the effective potential it means that the effective potential has to have an extremum. Taking into account the higher order corrections, with ϕ-dependent coefficients, the condition cannot be given only in terms of the effective potential and in fact one will have to put the condition directly on f which is the generalization of the leading order condition. Namely the function f has to have an extremum with respect to ϕ, *i.e.* $\partial_\phi f|_{\phi_0} = 0$ must have a solution. Therefore as far as the entropy computation is concerned we just need to extremize the entropy function and the entropy is given in terms of the value of entropy function at extremum.

We note, however, that the way the entropy function is defined might give an insight whether one needs to put another condition on f. In fact the function f is very similar to what is defined as the free energy for a system with gravitational interaction [41] in which one can identify the free energy with the Euclidean gravitational action times the temperature. If f could be interpreted as the free energy of the system, then one may want to assume $\partial_\phi^2 f|_{\phi_0} > 0$ to get the stable solution where the free energy is minimum. Otherwise the solution could be unstable which might mean there is not attractor behavior.[3] Note that in leading order this condition is equivalent to have minimum for the effective potential which is crucial to have attractor mechanism [17].

4. Exact solution

Let us now consider an explicit example in which the equations can exactly be solved. To be precise consider a system with one scalar and two gauge fields and the following potential

$$V_{\rm eff}(\phi) = e^{\alpha_1\phi}p_1^2 + e^{\alpha_2\phi}p_2^2. \tag{4.1}$$

For the 4-dimensional extremal black hole this potential have been studied in [17] where the authors have noticed that for the special case where $\alpha_1 = -\alpha_2$ the entropy is given by

$$S = 2\pi|p_1p_2|. \tag{4.2}$$

For $\alpha_1 = -\alpha_2 = 2$ it is in fact the supersymmetric black hole solution studied in [42] whose entropy is exactly (4.2). Note that although for generic values of α_1 and α_2 the solution is not supersymmetric, as long as $\alpha_1 = -\alpha_2$ the entropy is still given by (4.2). Therefore one might conclude that as far as the entropy of the black hole is concerned it is the attractor mechanism which plays the role which is the same for supersymmetric and non-supersymmetric cases.

Let us now consider higher order corrections to this entropy using the results we have presented in the previous section. To be concrete we shall consider the case where $g(\phi) = e^{\alpha_3\phi}$. In this case the critical value ϕ_0 is given by the solution of the following equation

$$e^{-2\alpha_2\phi_0} + \frac{4\tilde{\lambda}_2\alpha_3}{\alpha_2 p_1^2}e^{(\alpha_3-\alpha_2)\phi_0} - \frac{p_2^2}{p_1^2} = 0\,. \tag{4.3}$$

To proceed let us further assume that $\alpha_3 = \alpha_2$. For this case the entropy is given by

$$S = \pi|p_1p_2|\left((1-\frac{4\tilde{\lambda}}{p_2^2})^{1/2} + (1-\frac{4\tilde{\lambda}}{p_2^2})^{-1/2}\right) + 4\pi\tilde{\lambda}\left|\frac{p_1}{p_2}\right|(1-\frac{4\tilde{\lambda}}{p_2^2})^{-1/2}. \tag{4.4}$$

In the large p_2, p_1 limit, one may expand this expression to get

$$S = 2\pi|p_1p_2| + 4\pi\tilde{\lambda}\left|\frac{p_1}{p_2}\right| + \mathcal{O}(p_2^{-2}). \tag{4.5}$$

We note that if we had considered $\alpha_3 = -\alpha_2$, we would have gotten the same result as above except that the leading order is now proportional to $|p_2/p_1|$.

[3] Non-supersymmetric attractor mechanism in the presence of R^2 correction has recently been studied in [40].

Let us also study the five dimensional extremal black hole with the same potential as above, though here we will consider the case where $\alpha_1 = -2\alpha_2$. We shall first evaluate the entropy in the leading order which is given in terms of the effective potential $V_{\rm eff}(\phi_0)$ where ϕ_0 is given by

$$e^{\alpha_2\phi_0} = \left(\frac{\sqrt{2}p_1}{p_2}\right)^{2/3}, \tag{4.6}$$

and therefore we arrive at

$$S = \frac{3^{3/4}\pi^2}{2}\sqrt{\frac{p_1 p_2^2}{2}}. \tag{4.7}$$

This could be compared with 5-dimensional black hole studied in [28], of course we have an extra $3^{3/4}\pi/4$ factor. Indeed as far as the potential is concerned this case has the same potential structure as that considered in [28]. Therefore this model could be used to understand the five dimensional black hole better. We will come back to this point later.

On the other hand adding the higher order correction to the action such that $\lambda_2 = \tilde{\lambda}_2 e^{\alpha_3\phi}$ and using the correction we found for the entropy for the 5-dimensional black hole (3.9) and setting $\alpha_3 = \alpha_2$ one finds

$$S = (\frac{3^{3/4}\pi}{4})\left(2\pi\sqrt{\frac{p_1p_2^2}{2}}\right)\left[1 + 2^{8/3}\sqrt{3}\tilde{\lambda}\left(\frac{p_1}{p_2^4}\right)^{1/3} + \cdots\right]. \tag{4.8}$$

Higher order corrections for 5-dimensional black holes and black rings have recently been studied in [29]. Of course in this paper the authors have considered BPS solution, though in our case we have not assumed any supersymmetry. It is worth noting that although our solution is not supersymmetric, we get a correction to the entropy which has the same structure as that in BPS black hole studied in [29], namely setting $p_1 = 2p_2 = 2Q$, the above equation reads

$$S = (\frac{3^{3/4}\pi}{4})(2\pi\sqrt{Q^3})\left(1 + 2^3\sqrt{3}\frac{\tilde{\lambda}}{Q} + \cdots\right). \tag{4.9}$$

It would be interesting to further study this case and compare this non-supersymmetric model with the BPS black hole solution.

In general we can consider D-dimensional black hole solution with the above effective potential. In this case assuming $\alpha_1 = -(D-3)\alpha_2$, in leading order, one finds

$$S \sim (p_1 p_2^{D-3})^{1/(D-3)}. \tag{4.10}$$

5. Discussions

In this section we shall study different physical aspects of the black hole solutions we have considered in the previous sections. So far we have studied higher order corrections to the entropy of extremal black hole (2.5) by making use of the entropy function which can be defined as the Lagrangian density integrated over $(D-2)$-sphere. In this procedure one only needs to know the behavior of the solution near the horizon. Although the Lagrangian density is in general r-dependent function, the r is treated as a fixed parameter. Of course it will dropped in the final expression we have found for the entropy. In fact since the physics is governed by the behavior of the black hole solution in near horizon limit, one should set $r = r_H$ which can be found in terms of the effective potential in its extremum.

Of course it is not the only way to calculate the higher order corrections to the entropy of an extremal black hole with spherically symmetric metric. Actually it has been shown [43] (see also [39]) that for general relativity in $D \geq 3$ dimensions one can find an effective two dimensional theory which governs the conformal dynamics at the horizon and the entropy of the black hole is given by the central charge of the corresponding Virasoro algebra. It is interesting to compare this computations with what we have done in this paper.

To do that we start from the most general action we have considered in this paper and evaluate it for the ansatz (3.2) which leads to an effective two dimensional action such that at near horizon one finds

$$I = \frac{\Omega_{D-2}}{16\pi} \int d^2x\sqrt{-g}\Big\{ - \frac{(D-2)!\, V_{\rm eff}}{b^{D-2}} + \sum_{m=1}^{[D/2]} \frac{(D-2)!}{(D-2m)!}\lambda_m b^{D-2m} \times \left(\frac{(D-2m)(D-2m-1) - mb^2\mathcal{R}}{b^2}\right)\Big\}. \tag{5.1}$$

Here we have used the fact that the covariant derivative of different fields goes to zero near horizon. Following [43, 39] if we define

$$\varphi = \frac{2\Phi^2}{q\Phi_H}, \qquad \tilde{g}_{ab} = \frac{d\varphi}{db}e^{-2\varphi/q\Phi_H}g_{ab}, \qquad \Phi^2 = 2\frac{\Omega_{D-2}}{\kappa_D^2}\sum_{m=1}^{[D/2]} m\lambda_m\frac{(D-2)!}{(D-2m)!}\, b^{D-2m}, \tag{5.2}$$

the above two dimensional effective action reads

$$I = \int d^2x\sqrt{-g}\left(\frac{1}{2}(\partial\varphi)^2 + \frac{q}{4}\Phi_H\varphi\mathcal{R} + U(\varphi)\right). \tag{5.3}$$

On the other hand for the extremal black hole we are interested in the two dimensional part of the metric can be written as $ds^2 = a^2(z)(-dt^2 + dz^2)$ where $z = \int dr/a^2(r)$. It is easy to see that for the solutions we have been discussing in which $a^2(r)$ has double zero at horizon, $z \to -\infty$ at $r \to r_H$. Now the crucial observation [43, 39] is that the trace of energy-momentum tensor of this two dimensional system vanishes near the horizon and therefore the theory of the scalar field φ approaches a CFT near the horizon with central charge $c = 3\pi q^2\Phi_h^2$. The entropy of the black hole is also given by

$$S = 2\pi\Phi_H^2 = \frac{\Omega_{D-2}}{4}\sum_{m=1}^{[D/2]} m\lambda_m\frac{(D-2)!}{(D-2m)!}\, r_h^{D-2m}. \tag{5.4}$$

In particular for $D = 5$ case one finds $S = \frac{\pi^2}{2}(r_H^3 + 12\lambda r_H)$ which upon using the fact that in this case $r_H = V_{\rm eff}^{1/4}$ one get the same result as (3.9). Writing the entropy in terms of Φ_H is instructive. Actually Φ^2 as a function of r whose value at horizon gives the central charge of the CFT, can be treated as a parameter which tells us how near we are to the horizon, where the theory becomes a CFT. Therefore it is tempting to consider it as a C-function

$$C = \Phi^2 = \frac{\Omega_{D-2}}{8\pi} \sum_{m=1}^{[D/2]} m\lambda_m \frac{(D-2)!}{(D-2m)!}\, r^{D-2m}. \tag{5.5}$$

In fact for $D = 4$ and in the leading order, $m = 1$, it is the C-function introduced in [22]. Using the same argument as that in [22] one can see that it is a monotonically decreasing function. Moreover as we have seen at the fixed point (horizon) it is the central charge of the corresponding CFT.

So far we have only considered the cases where the black hole has no electric charges. We note however that the procedure can be generalized for the case where we have electric charges as well. For this case one may still concentrate at the near horizon solution where r is kept fixed and indeed equal to the radius of the horizon and therefore the electric gauge fields can be chosen as $F^a_{rt} = \frac{4\pi}{\Omega_{D-2}} e^a$. Using the same argument as [23] it is now easy to see that in this case the entropy can be found from the Legendre transformation of f as follows

$$S = \frac{4\pi}{a^2(r)''}\left(e^a \frac{\partial f}{\partial e^a} - f\right), \tag{5.6}$$

and the electric charges of the black hole is given by $Q_a = \frac{\partial f}{\partial e^a}$ which for our ansatz and the action (2.2) it reads

$$Q_a = \frac{4\pi}{\Omega_{D-2}} \frac{v_2^{(D-2)/2} b^{D-2}}{v_1} f^{(e)}_{ab} e^b. \tag{5.7}$$

By making use of this expression and plugging it into the equation (5.6), one can see that the entropy is given by equation (2.10) with f defined as (3.4) but with a modified effective potential. The modified effective potential is given by

$$V_{\rm eff} = f^{(m)}_{\alpha\beta} p^\alpha p^\beta + \frac{2}{(D-2)!} {f^{(e)}}^{ab} Q_a Q_b\ . \tag{5.8}$$

where ${f^{(e)}}^{ab}$ is the inverse of ${f^{(e)}}_{ab}$, of course we assume that it is invertible. This procedure can also be used when we have axionic coupling as well.

Finally let us consider the five dimensional supersymmetric black hole studied in [28] in more detail. The corresponding five dimensional action which can be obtained by compactification of type II string theory on $K3 \times S^1$ in the Einstein frame is given by

$$I = \frac{1}{16\pi} \int d^4x \sqrt{-G} \left(R - \frac{4}{3} (\partial\phi)^2 - \frac{e^{-4\phi/3}}{4} H^2 - \frac{e^{2\phi/3}}{4} F^2 \right), \tag{5.9}$$

where F is a RR 2-form field strength and H is a 2-form axion field strength arising from the NS-NS 3-form with one component tangent to the S^1. We shall consider the following ansatz for the near horizon solution of the 5-dimensional black hole

$$\begin{aligned} ds^2 &= v_1 \left(-a^2(r) dt^2 + \frac{dr^2}{a^2(r)} \right) + v_2 r^2 d\Omega_3, \qquad e^{2\phi/3} = u, \\ F_{tr} &= e_F, \qquad H_{tr} = \frac{4}{\pi} e_H\,, \end{aligned} \tag{5.10}$$

in which the entropy function, f, reads

$$f = \frac{\pi}{8} v_1 v_2^{3/2} r^3 \left(\frac{6v_1 - v_2 r^2 a^2(r)''}{v_1 v_2 r^2} + \frac{8e_H^2}{\pi^2 v_1^2 u^2} + \frac{e_F^2 u}{2v_1^2} \right). \tag{5.11}$$

Therefore the black hole electric charges are given by

$$Q_F = \frac{\partial f}{\partial e_F} = \frac{\pi}{8} \frac{v_2^{3/2} r^3}{v_1} u e_F, \qquad Q_H = \frac{\partial f}{\partial e_H} = \frac{2}{\pi} \frac{v_2^{3/2} r^3}{v_1 u^2} e_H, \tag{5.12}$$

Now one needs to extremize the entropy function with respect to v_1, v_2 and u. Doing so one finds

$$v_1 = \frac{a^2(r)''}{8} \left(\frac{8Q_H Q_F^2}{\pi^2} \right)^{1/3}, \qquad v_2 = \frac{1}{r^2} \left(\frac{8Q_H Q_F^2}{\pi^2} \right)^{1/3}, \qquad u^3 = \frac{1}{2} \left(\frac{4Q_F}{\pi Q_H} \right)^2, \tag{5.13}$$

which together with (5.6) can be used to find the entropy as follows

$$S = 2\pi \sqrt{\frac{Q_H Q_F^2}{2}}. \tag{5.14}$$

The radius of horizon is also found as

$$r_H = \left(\frac{8Q_H Q_F^2}{\pi^2} \right)^{1/6}. \tag{5.15}$$

Essentially the result is the same as what we have considered in previous section up to a numerical factor. To understand the origin of this numerical mismatch one may use electric-magnetic duality to write the gauge fields as follow

$$F_3 = \frac{8}{\pi} Q_f \sqrt{\omega_3}, \qquad H_3 = 2Q_H \sqrt{\omega_3}. \tag{5.16}$$

We note that in comparison with (2.5) one needs to rescale p^α proprely. Doing so and going through the computations we find the correct numerical factors. Using the R^2 correction to the effective action one can also find the corrections to the entropy.

Acknowledgments

We would like to thank B. Chandrasekhar, A. Ghodsi and S. Parvizi for discussions on the related topics.

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